# Chapter 7_L14

- Rate equations

✓ Four-level laser

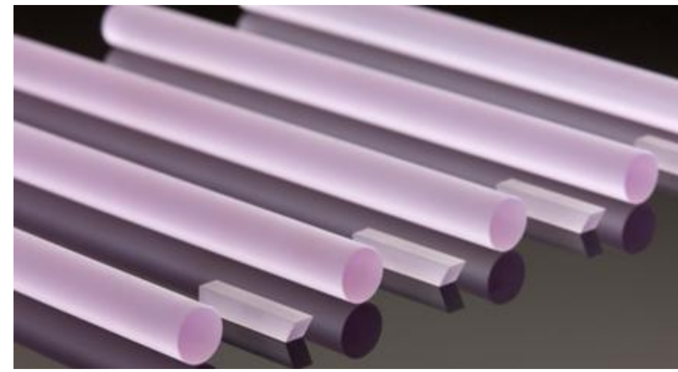
✓ Three-level laser

# Rate equations



- Rate equations are a set of coupled differential equations, describing the temporal (and spatial) behavoirs of the carriers (populations) and the photons (lasing power). It is a phenomenonlogical description of the Bloch-Maxwell's equations

- Two main types of lasers:



✓ Four-level lasers: Ionic crystal lasers like $Nd^{3+}$:YAG (Neodymium-doped Yttrium Aluminium Garnet 掺钕钇铝石榴石 1064 nm); Gas lasers like $CO_2$ (10.6 um), He-Ne (632.5 nm)

✓ Three-level lasers: fiber lasers like Er (铒1550 nm), Yb (镱1030 nm) , Ho (钬2000 nm) , Tm (铥2000 nm) doped fiber lasers

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# Four-level laser



- Assumptions:

  The carrier density (population) is uniform inside the gain medium

  The photon density (electric field) is uniform inside the cavity

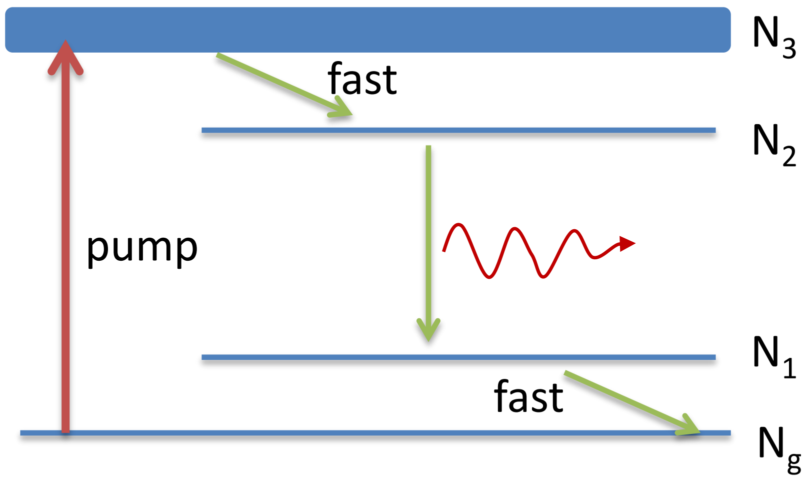
  The light volume is the same as the gain medium volume

  Only one mode oscillates in the cavity

  Then, the rate equation is space independent.

- The lifetimes of level 3 and level 1 is very fast

$$\boxed{N_3 \approx 0, \quad N_1 \approx 0}$$




- The laser emission occurs at level 3→ level 2

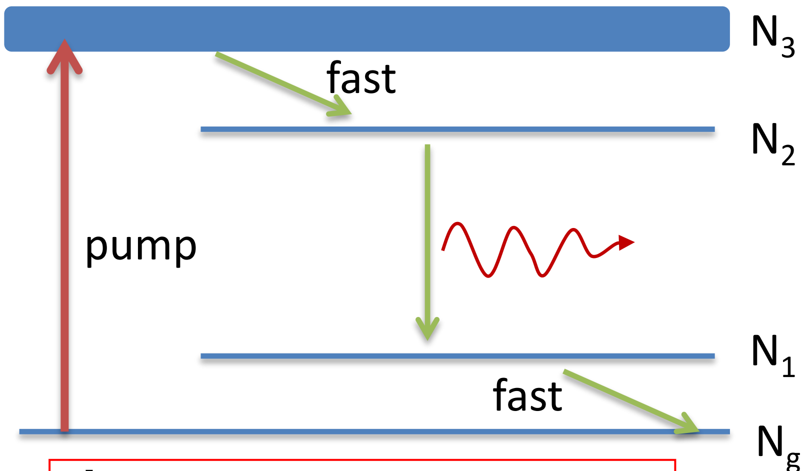
The population inversion

$\Delta N \approx N_2$

The gain

$g = \sigma \Delta N \approx \sigma N_2$

# Four-level laser




$$W\left(N_2-N_1\right)=W\Delta N=\sigma \mathrm{F}\Delta N$$
$$=\sigma N_P \mathrm{v_g}\Delta N \approx \sigma N_P \mathrm{v_g} N_2$$
$$=\mathrm{v_g} g N_P$$

The rate equations

$$\frac{dN_2}{dt}=R_p-W\left(N_2-N_1\right)-\frac{N_2}{\tau_{sp}}$$

$$\frac{dN_P}{dt}=W\left(N_2-N_1\right)-\frac{N_P}{\tau_p}+\beta\frac{N_2}{\tau_{sp}}$$

$$N_3 \approx 0$$

$$N_1 \approx 0$$

$$N_g \approx const$$

$R_p$ is the pump rate

$\beta$ is the spontaneous emission factor

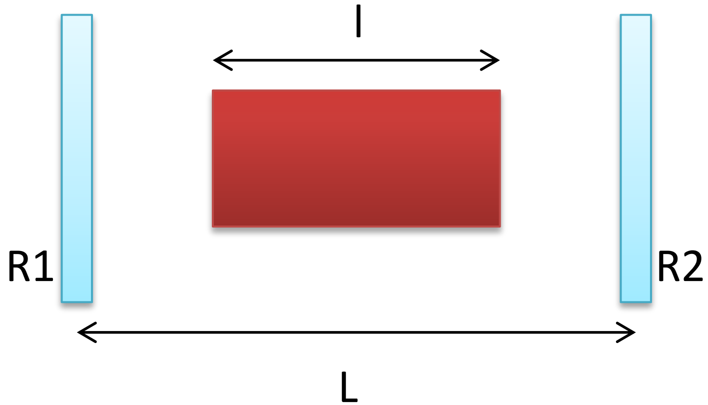
The rate equations

$$\frac{dN_2}{dt}=R_p-\mathrm{v_g} g N_P-\frac{N_2}{\tau_{sp}}$$

$$\frac{dN_P}{dt}=\mathrm{v_g} g N_P-\frac{N_P}{\tau_p}+\beta\frac{N_2}{\tau_{sp}}$$

$\beta\frac{N_2}{\tau_{sp}}$ is small, and is negligible in most discussions.

# Four-level laser




- Optical length

$$L_o = (L - l) + n_r l$$

- Round-trip time

$$\Delta t = 2\frac{L_o}{c}$$

- Loss coefficient

$$\alpha_T = \alpha_i + \frac{1}{2L}\ln\frac{1}{R_1 R_2}$$

- gain coefficient

$$g = \sigma \Delta N$$

- Photon lifetime

$$\tau_p = \frac{1}{v_g \alpha_T}$$

- Some relations

Photon flux ($\mathrm{cm^{-2}s^{-1}}$)

$F = v_g N_P$

Energy density ($\mathrm{J/cm^3}$)

$\rho = N_P h\upsilon$

Light intensity($\mathrm{mW/cm^2}$)

$I = v_g N_P h\upsilon$

$= Fh\upsilon = \rho v_g$

# The output power



- The energy inside the cavity

$$E = N_P h\upsilon V_p$$

- The energy output (loss) rate of the two mirrors

$$\frac{1}{\tau_m} = v_g \alpha_m = v_g \frac{1}{2L} \ln \frac{1}{R_1 R_2}$$

$$\text{If } R_1 = R_2 \Rightarrow$$

$$\frac{1}{\tau_m} = v_g \frac{1}{L} \ln \frac{1}{R}$$

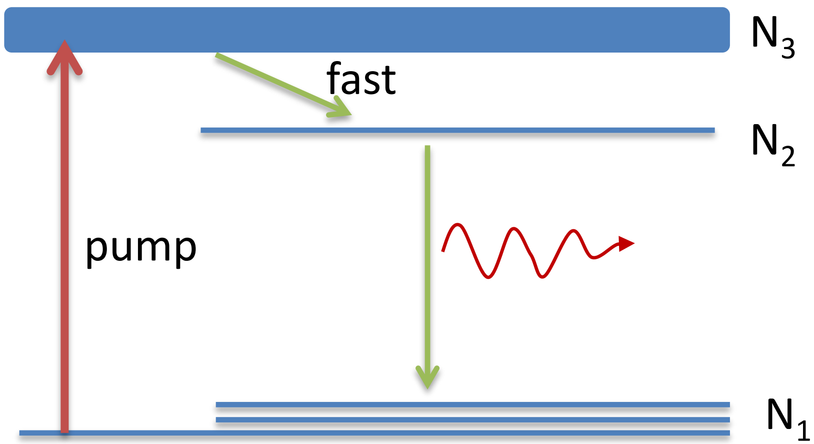
- The total output power from the two mirrors is

$$\boxed{P_{out} = \frac{E}{\tau_m} = \left(N_P h\upsilon V_p\right)\left(v_g \alpha_m\right)}$$

Examples 7.1

# (Quasi-) three-level laser






- In a (quasi-) three-level laser, the lower laser level is the ground level, or a sub-level of the ground level.
  - ✓All the sublevels are strongly coupled and hence in thermal equilibrium.

$$\frac{d\Delta N}{dt} = \frac{d}{dt}\left(N_2 - \frac{g_2}{g_1}N_1\right) = \left(1+\frac{g_2}{g_1}\right)\frac{dN_2}{dt}$$

$$\left(1+\frac{g_2}{g_1}\right)\frac{N_2}{\tau_{sp}} = \frac{1}{\tau_{sp}}\left[N_2 + \frac{g_2}{g_1}(N_t - N_1)\right] = \frac{1}{\tau_{sp}}\left(\Delta N + \frac{g_2}{g_1}N_t\right)$$

The rate equations

$$\frac{dN_2}{dt} = R_p - \mathrm{v}_g g N_P - \frac{N_2}{\tau_{sp}}$$

$$\frac{dN_P}{dt} = \mathrm{v}_g g N_P - \frac{N_P}{\tau_p} + \beta\frac{N_2}{\tau_{sp}}$$

$$N_1 = N_t - N_2$$

$$N_3 \approx 0$$

The rate equations

$$\frac{d\Delta N}{dt} = \left(1+\frac{g_2}{g_1}\right)R_p - \left(1+\frac{g_2}{g_1}\right)\mathrm{v}_g g N_P - \frac{1}{\tau_{sp}}\left(\Delta N + \frac{g_2}{g_1}N_t\right)$$

$$\frac{dN_P}{dt} = \mathrm{v}_g g N_P - \frac{N_P}{\tau_p} + \beta\frac{1}{\tau_{sp}}\left(\Delta N + \frac{g_2}{g_1}N_t\right)$$

- gain coefficient

$$g = \sigma\Delta N = \sigma\left(N_2 - \frac{g_2}{g_1}N_1\right)$$

## Lasing threshold and power

- ✓ Four-level laser
- ✓ Three-level laser

# Four-level laser

- The carrier lifetime

$$\frac{1}{\tau} = \frac{1}{\tau_{sp}} + \frac{1}{\tau_{nr}}$$

- Condition for possible continuous-wave lasing

$$\boxed{\tau_2 > \tau_1}$$

- Ignore the spon. emission

$$\boxed{\begin{aligned} &\frac{dN_P}{dt} = \mathrm{v_g} g N_P - \frac{N_P}{\tau_p} \Rightarrow \\ &N_p(t) = N_p(0)\exp\left(\mathrm{v_g} g - \frac{1}{\tau_p}\right)t \end{aligned}}$$

- To achieve lasing,

$$\boxed{N_p(t) > N_p(0) \Rightarrow \mathrm{v_g} g \geq \frac{1}{\tau_p}}$$

- Thus, the lasing threshold is

$$\boxed{g_{th} = \frac{1}{\mathrm{v_g}\tau_p} \Rightarrow \Delta N_{th} \approx N_{2th} = \frac{1}{\sigma \mathrm{v_g} \tau_p}}$$

The rate equations

$$\frac{dN_2}{dt} = R_p - \mathrm{v_g} g N_P - \frac{N_2}{\tau_{sp}}$$

$$\frac{dN_P}{dt} = \mathrm{v_g} g N_P - \frac{N_P}{\tau_p} + \beta \frac{N_2}{\tau_{sp}}$$

# Below the threshold



- Below the threshold, the stimulated emission is negligible (photon number is small), and there is only spontaneous emission, then the rate equations become

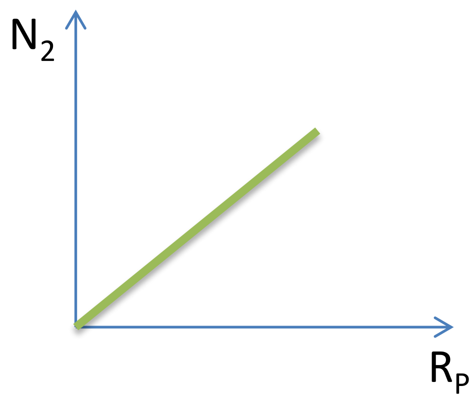
$$\boxed{\begin{aligned} \frac{dN_2}{dt} &= R_p - \frac{N_2}{\tau_{sp}} \\ \frac{dN_P}{dt} &= -\frac{N_P}{\tau_p} + \beta\frac{N_2}{\tau_{sp}} \end{aligned}}$$

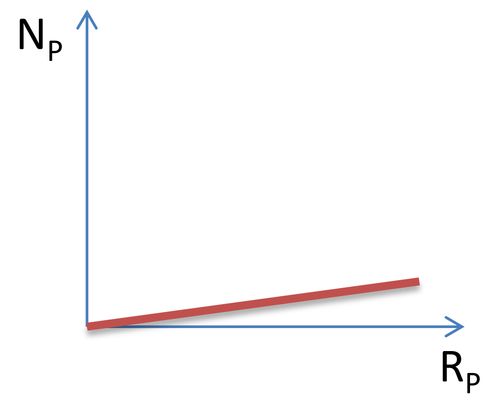
- Under steady-state condition

$$\boxed{\frac{dN_2}{dt} = 0; \quad \frac{dN_P}{dt} = 0}$$

- The steady-state solutions are

$$\boxed{\begin{aligned} N_2 &= R_p\tau_{sp} \\ N_P &= \beta R_p\tau_p \end{aligned}}$$




- Both the carrier density and the photon density increase linearly with the pump rate

The rate equations

$$\frac{dN_2}{dt} = R_p - v_g g N_P - \frac{N_2}{\tau_{sp}}$$

$$\frac{dN_P}{dt} = v_g g N_P - \frac{N_P}{\tau_p} + \beta\frac{N_2}{\tau_{sp}}$$

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# (Well) above the threshold



- (Well) above the threshold, the stimulated emission dominates, and the spontaneous emission is negligible, then the rate equations become

$$\frac{dN_2}{dt}=R_p-\mathrm{v}_g gN_P-\frac{N_2}{\tau_{sp}}$$

$$\frac{dN_P}{dt}=\mathrm{v}_g gN_P-\frac{N_P}{\tau_p}$$

- The steady-state solutions are

$$\frac{dN_P}{dt}=0\Rightarrow$$

$$g=\frac{1}{\mathrm{v}_g\tau_p};\ N_2=\frac{1}{\sigma\mathrm{v}_g\tau_p}\Rightarrow$$

$$g=g_{th};\ N_2=N_{2th}$$

- **Gain clamping**:

Above the threshold, the gain does not increase with the pump rate, but remains the same as that at the threshold. (carrier population clamping)

$$\frac{dN}{dt}=0\Rightarrow$$

$$N_P=\tau_p\left(R_p-\frac{N_{2\mathrm{th}}}{\tau_{sp}}\right)$$

- The photon density increases linearly with the pump rate.
- The threshold pump rate is ($N_p>=0$)

$$R_{\mathrm{pth}}=\frac{N_{2th}}{\tau_{sp}}=\frac{1}{\sigma\mathrm{v}_g\tau_p\tau_{sp}}$$

The rate equations

$$\frac{dN_2}{dt}=R_p-\mathrm{v}_g gN_P-\frac{N_2}{\tau_{sp}}$$

$$\frac{dN_P}{dt}=\mathrm{v}_g gN_P-\frac{N_P}{\tau_p}+\beta\frac{N_2}{\tau_{sp}}$$

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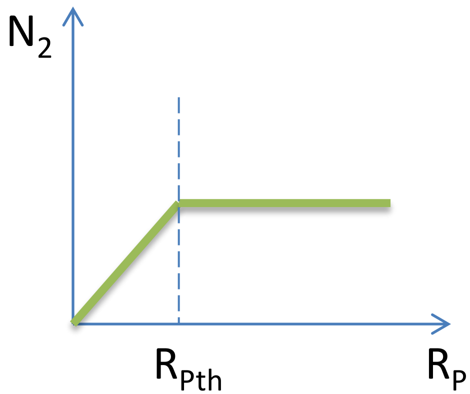
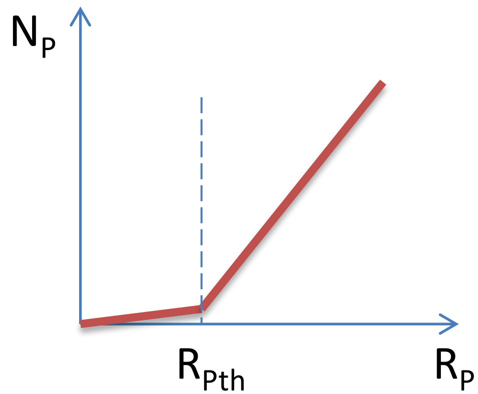
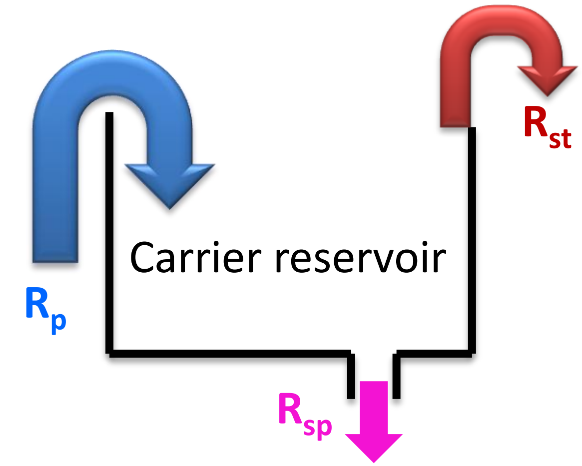
# Below and above the threshold






- Below the threshold

$$N_2 = R_p \tau_{sp}$$
$$N_P = \beta R_p \tau_p$$

- At the threshold

$$\mathrm{v}_g g_{th} = \frac{1}{\tau_p}$$
$$N_{2th} = \frac{1}{\sigma \mathrm{v}_g \tau_p}$$
$$R_{pth} = \frac{N_{2th}}{\tau_{sp}}$$
$$N_{pth} = \beta R_p \tau_p \approx 0$$

- Above the threshold

$$N_2 = N_{2th}$$
$$N_P = \tau_p \left(R_p - R_{pth}\right)$$

The rate equations

$$\frac{dN_2}{dt} = R_p - \mathrm{v}_g g N_P - \frac{N_2}{\tau_{sp}}$$
$$\frac{dN_P}{dt} = \mathrm{v}_g g N_P - \frac{N_P}{\tau_p} + \beta \frac{N_2}{\tau_{sp}}$$

# The output power



- The photon density

$$\boxed{N_P = \tau_p \left(R_p - R_{pth}\right)}$$

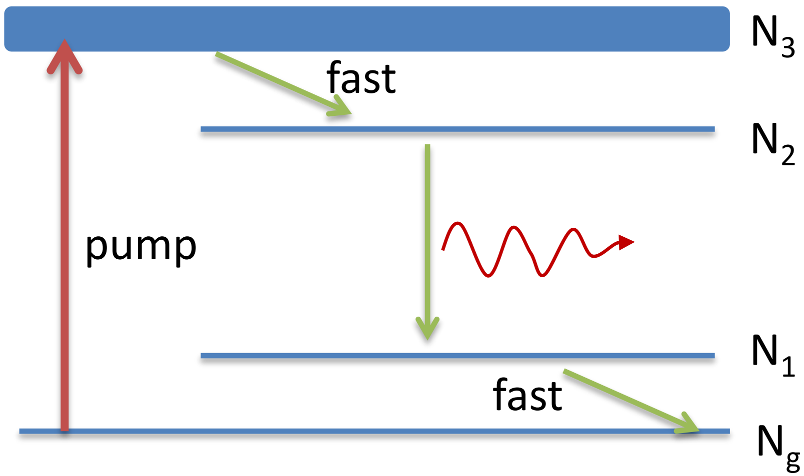
$$\boxed{\begin{aligned} N_P &= R_{pth}\tau_p \left(\frac{R_p}{R_{pth}} - 1\right) \\ &= \frac{1}{\sigma \mathrm{v}_g \tau_{sp}} \left(\frac{R_p}{R_{pth}} - 1\right) \end{aligned}}$$

- The output power

$$\boxed{\begin{aligned} P_{out} &= \left(N_p h\upsilon V_p\right)\left(v_g \alpha_m\right) \\ &= \tau_p h\upsilon V_p \left(v_g \alpha_m\right)\left(R_p - R_{pth}\right) \end{aligned}}$$

$$\boxed{\begin{aligned} P_{out} &= \left(N_p h\upsilon V_p\right)\left(v_g \alpha_m\right) \\ &= \alpha_m \frac{h\upsilon}{\sigma \tau_{sp}} V_p \left(\frac{R_p}{R_{pth}} - 1\right) \\ &= \alpha_m I_s V_p \left(\frac{R_p}{R_{pth}} - 1\right) \end{aligned}}$$

# The output power




- For the optical pump, the pump power $P_p$

$$P_p = R_p V h\nu_p = \left(\frac{dN_2}{dt}\right)_p V h\nu_p$$

$$P_{out} = \alpha_m I_s V_p \left(\frac{R_p}{R_{pth}} - 1\right) = \alpha_m I_s V_p \left(\frac{P_p}{P_{pth}} - 1\right)$$

- For the electrical pump, the pump current *I*

$$R_p = \frac{I}{qV}$$

$$P_{out} = \alpha_m I_s V_p \left(\frac{R_p}{R_{pth}} - 1\right) = \alpha_m I_s V_p \left(\frac{I}{I_{th}} - 1\right)$$

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# The output power



- The laser slope efficiency: the output laser power versus the input light power or injected current

$$\eta_{so} = \frac{dP_{out}}{dP_p} = \frac{\alpha_m I_s V_p}{P_{pth}} \quad \text{(W/W)}$$

$$\eta_{se} = \frac{dP_{out}}{dI} = \frac{\alpha_m I_s V_p}{I_{th}} \quad \text{(W/A)}$$

- The laser quantum efficiency: ratio of output photon numbers and input photon/ electron numbers

$$\eta_{qo} = \frac{P_{out} / hv}{P_p / hv_p}$$

$$\eta_{qe} = \frac{P_{out} / hv}{I / q}$$

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# (Quasi-) three-level laser



- The threshold population inversion (ignore spon. emission) $\boxed{\frac{dN_p}{dt}=0}$

$$\boxed{g_{th}=\frac{1}{v_g\tau_p};\ \Delta N_{th}=\frac{1}{\sigma v_g\tau_p}}$$

$$\boxed{N_{2th}=\frac{N_t+\Delta N_{th}}{2}\approx\frac{N_t}{2}}$$

- The threshold pump rate $\boxed{N_p=0;\ \frac{d\Delta N}{dt}=0}$

$$\boxed{R_{pth}=\frac{1}{1+g_2/g_1}\frac{1}{\tau_{sp}}\left(\Delta N_{th}+\frac{g_2}{g_1}N_t\right)}$$

- The photon density $\boxed{\frac{dN_p}{dt}=0;\ \frac{d\Delta N}{dt}=0}$

$$\boxed{N_P=\tau_p\left(R_p-R_{pth}\right)}$$

The rate equations

$$\frac{d\Delta N}{dt}=\left(1+\frac{g_2}{g_1}\right)R_p-\left(1+\frac{g_2}{g_1}\right)\mathrm{v}_g gN_P-\frac{1}{\tau_{sp}}\left(\Delta N+\frac{g_2}{g_1}N_t\right)$$

$$\frac{dN_P}{dt}=\mathrm{v}_g gN_P-\frac{N_P}{\tau_p}+\beta\frac{1}{\tau_{sp}}\left(\Delta N+\frac{g_2}{g_1}N_t\right)$$

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# Optimum coupling (max power)

- The mirror output coupling (loss)

$$\alpha_m = \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)$$

If $R_1 = R_2$

$$\alpha_m = \frac{1}{L}\ln\left(\frac{1}{R}\right)$$

✓ When T increases, on one hand, the output power tends to increase; on the other hand, the total photon number in the cavity tends to decrease.

- For the four-level lasers

$$R_{pth} = \frac{1}{\sigma \mathrm{v}_g \tau_p \tau_{sp}}$$

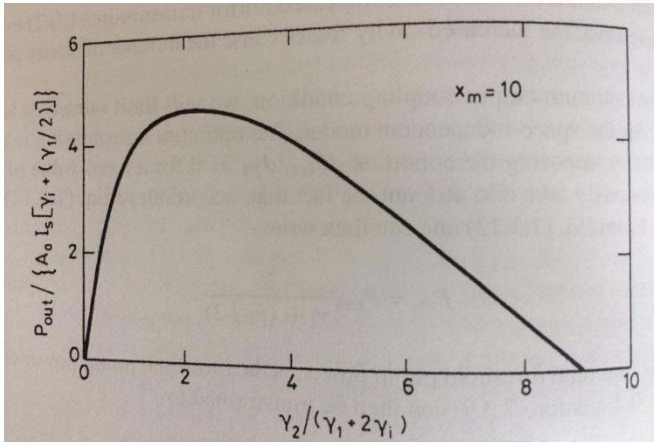
$$\tau_p = \frac{1}{\mathrm{v}_g(\alpha_i + \alpha_m)}$$

$$P_{out} = \tau_p h\upsilon V_p (v_g \alpha_m)(R_p - R_{pth}) = \alpha_m h\upsilon V_p \left(\frac{R_p}{\alpha_i + \alpha_m} - \frac{1}{\sigma\tau_{sp}}\right)$$

- For a fixed pump rate, the maximum power is at

$$\frac{d}{dt}P_{out} = 0 \Rightarrow$$

$$\alpha_m = \sqrt{R_p \sigma \tau_{sp} \alpha_i} - \alpha_i$$

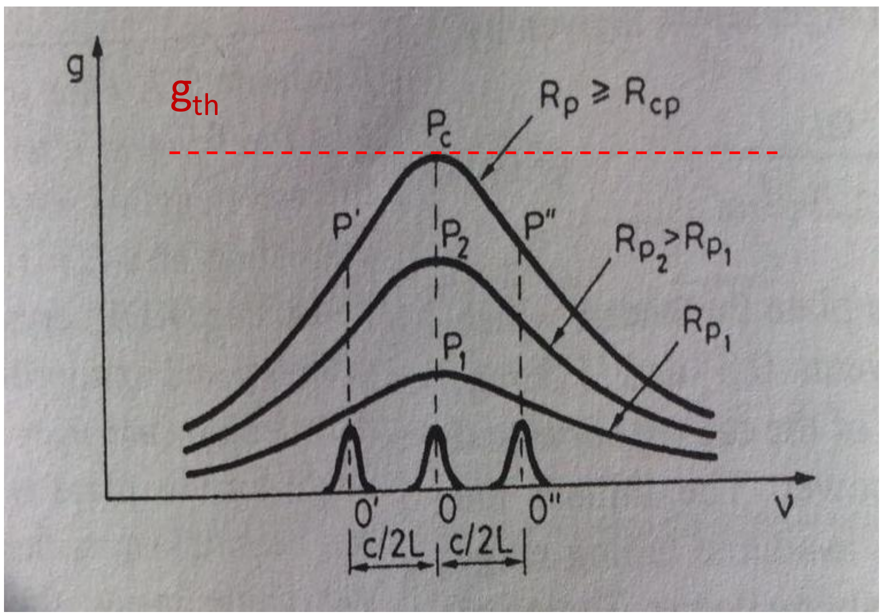


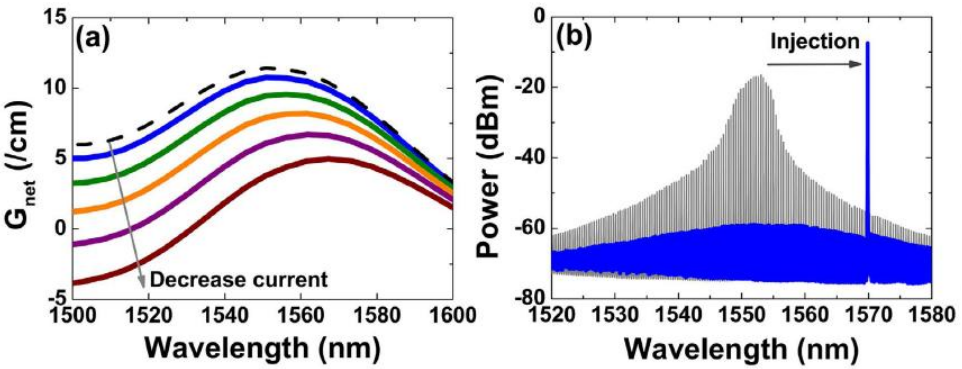
- Multimode oscillation

# Multimode (longitudinal) oscillation



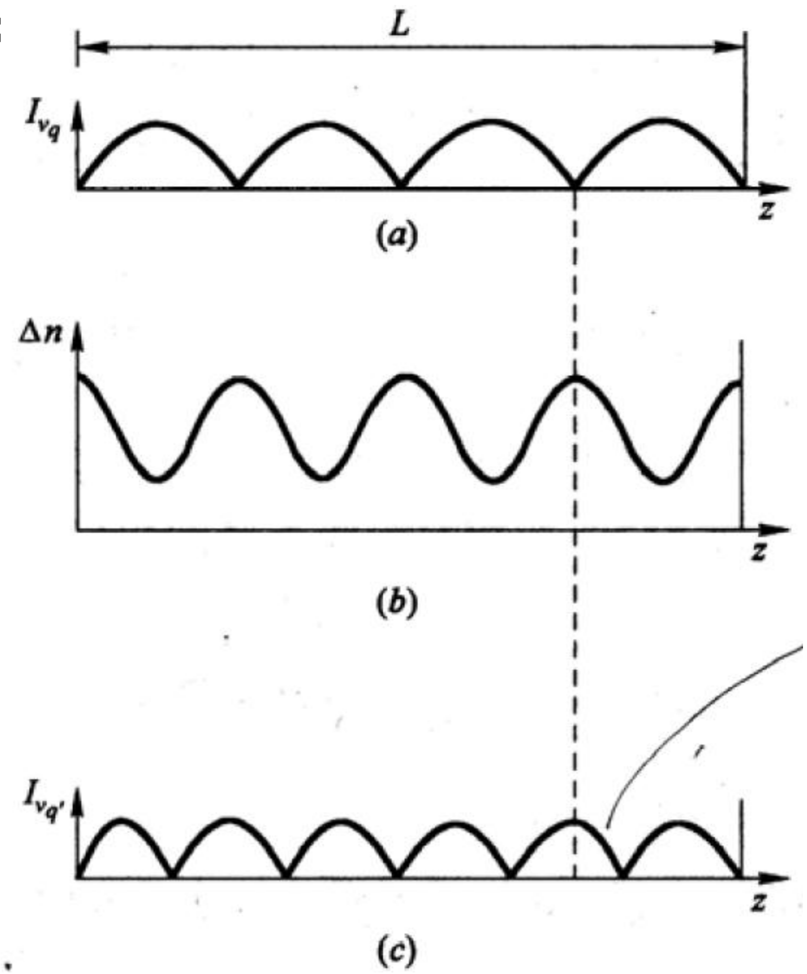
- Lasers usually oscillate on many modes, due to the usual wide gain profile, much broader than the mode spacing (FSR).
- **Homogeneous broadening medium** only oscillates on one mode at the gain peak, due to the gain clamping effect.



# Multimode (longitudinal) oscillation

- In practice, in **homo. broadening medium**, there are several modes oscillating around the gain peak **when the pumping is high enough.**




- Electric fields are standing waves in the cavity.
- For spatial points at extrema of the electric field, the population inversion are clamped.
- For spatical points at zeros of the electric field, the population inversion remains increasing.
- Thus, there will be holes formed at different positions of the gain medium. This is called **spatial hole burning**.

# Multimode (longitudinal) oscillation



- In the **homogeneous broadening medium**, due to the **spatial hole burning**, different longitudinal mode can use carriers at different places, so several modes around the gain peak can oscillates. These modes competes for the strongest lasing, and is called **(spatial) competition of (longitudinal) modes,** which leads to **the lasing power fluctuations.**

✓ In homo. gas medium, carriers moves fast in the space, spatial holes can hardly formed, so single mode can be obtained.

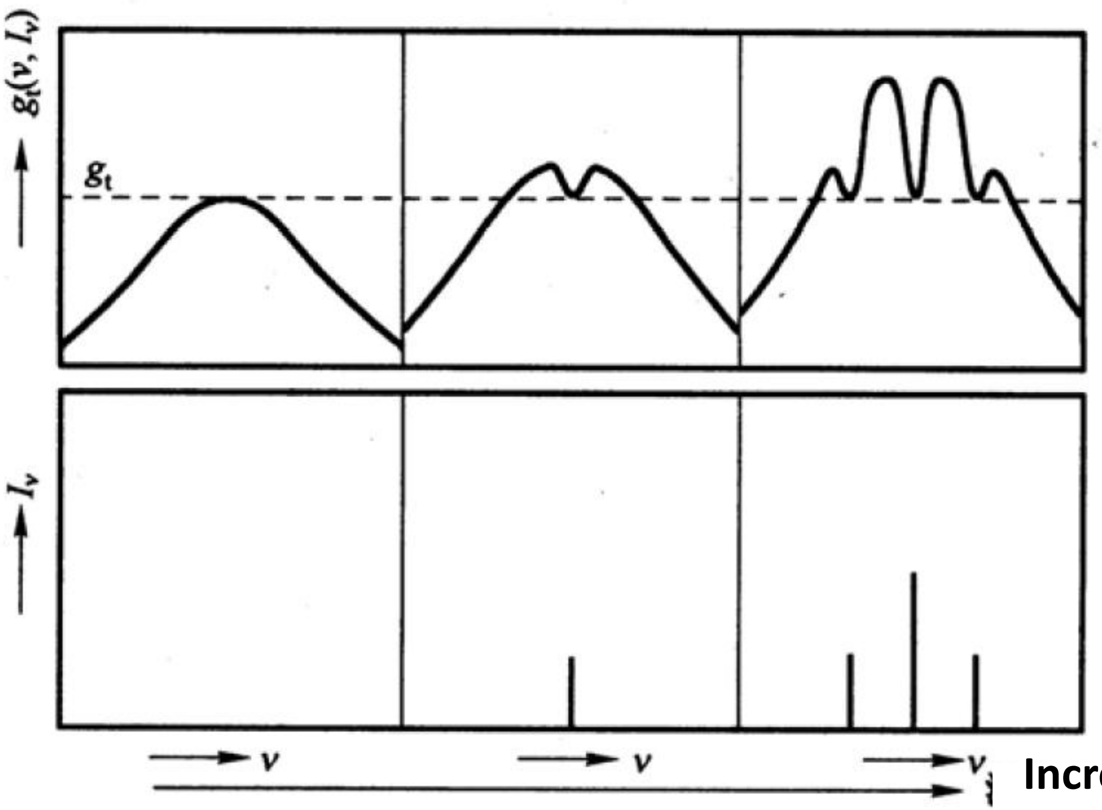
✓ In homo. solid medium, carriers moves slow in the space, spatial holes will be formed, and multimodes around the gain peak can be obtained.

✓ In ring cavity with isolator, the wave is traveling rather than standing, so no spatial hole burning effect, and single mode is formed.

# Multimode (longitudinal) oscillation



❑ **Inhomogeneous broadening medium** oscillates on many modes within in the gain profile, which are independent with each other. **Spectral hole burning** will be formed on the gain spectrum.




**Increase pump**

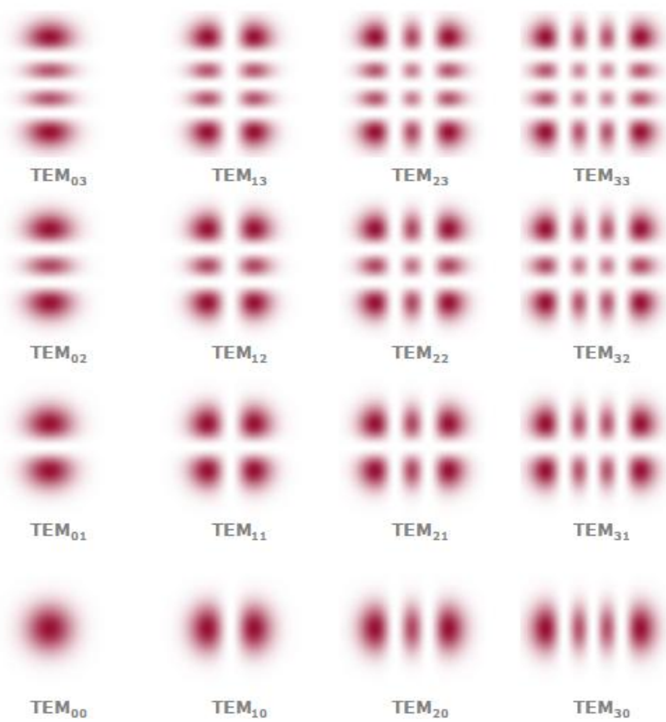
✓ If the mode spacing is too small, the holes will have some overlap with each other, leading to the mode competition due to the sharing of the same group carriers.

# Multimode (transverse) oscillation



- Different **transverse mode** has different diffraction loss, leading to different values of threshold. TEM00 has the lowest loss and thus oscillates firstly.

- The electric field distribution of transverse mode is not uniform, leading to the transverse spatial hole burning. Different transverse mode has different electric field distribution, and use different spatial carriers. When the pump is strong enough, there will be many modes oscillating simultaneously.

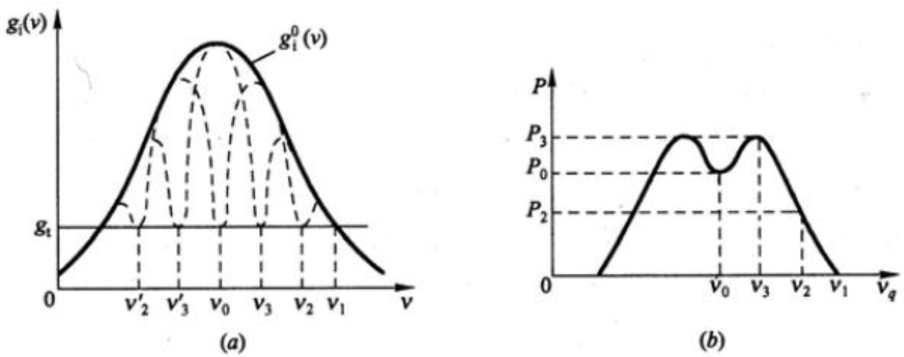


# Lamb dip



❑ In the laser with Doppler inhomo. broadening medium, the output power increases when the longitudinal mode frequency is tuned towards the central frequency (gain peak). However, when the mode frequency is close to the central frequency, the power will decrease instead. Then, a dip is form on the power spectrum, which is called Lamb dip.

✓ When the mode frequency $v_m$!=$v_0$, the light interacts with two groups of carriers with velocity $+v_z$ and $-v_z$. Both groups contribute to the lasing power.




✓ When the mode frequency $v_m=v_0$ , only one group of carrier with velocity $v_z$=0 interacts with the light, and contribute to the lasing power. Although the gain is highest, but the contributed carriers are the smallest. Therefore, the lasing power decreases, leading to the Lamb dip on the power spectrum.

✓ Lamb dip is a famous technique to stablize the laser frequency.

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## ❑ Single mode selection

- ✓ Transverse mode selection
- ✓ Longitudinal mode selection

# Single transverse mode selection



- TEM00 mode is usually desirable. The selection basis is that
  - ✓ The diffraction loss (δ) has an enough portion in comparison with other losses (α).
  - ✓ The diffraction loss increases with the order of the transverse mode.
- The mode selection requirement is

$$g^{00} > \alpha_T{}^{00} \text{ and } g^{01} < \alpha_T{}^{01},\ g^{10} < \alpha_T{}^{10}$$

- For the same diffraction loss portion, the larger the diffraction loss difference ( $\delta_{10}/\delta_{00}$) between the high-order mode and the fundamental mode, the easier the mode selection. (Note: absolute loss value must be considered)

$$\alpha = 20,$$
$$\delta_{00} = 0.1,\ \delta_{01} = 1,$$
$$\alpha_T^{00} = 20.1,\ \alpha_T^{01} = 21;$$

$$\alpha = 10,$$
$$\delta_{00} = 10,\ \delta_{01} = 15,$$
$$\alpha_T^{00} = 20,\ \alpha_T^{01} = 25;$$

$$\alpha = 10,$$
$$\delta_{00} = 10,\ \delta_{01} = 30,$$
$$\alpha_T^{00} = 20,\ \alpha_T^{01} = 40;$$

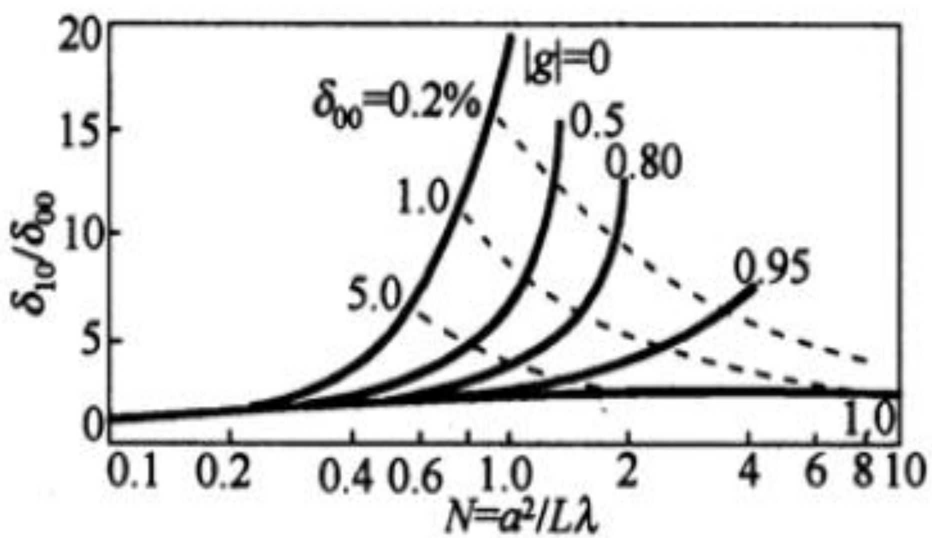
# Single transverse mode selection



✓ The diffraction loss decreases with Fresnel number, while the mode discrimination increases.

✓ The confocal cavity has the lowest loss, but the largest discrimination ability. The concentric cavity and the plane cavity have the highest loss, but the smallest dscrimination ability.

✓ In practice, it is easier to use the concentric and plane cavities to select the fundamental mode, due to the high mode losses.

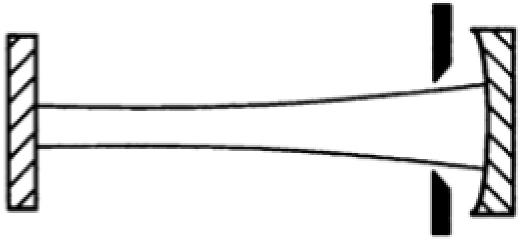



Mode discrimination versus Fresnel number

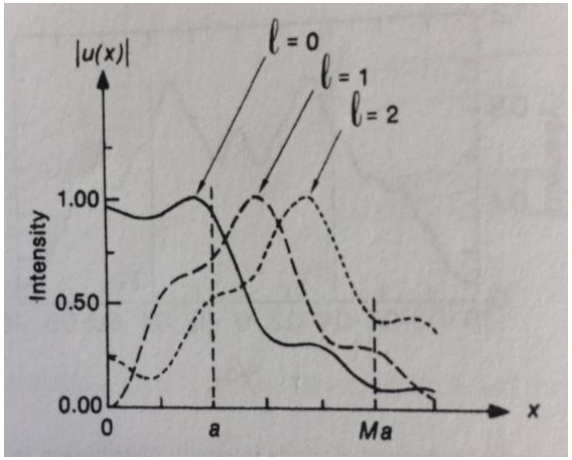
# Single transverse mode selection



- Design a proper aperture inside the cavity



- Design a proper parameter g and Fresnel number N of the cavity
- Design a proper unstable cavity




- Slight adjust the parallelization of mirrors

✓ For plane cavity, TEM00 will suffer the largest loss, and high-order modes oscillate.

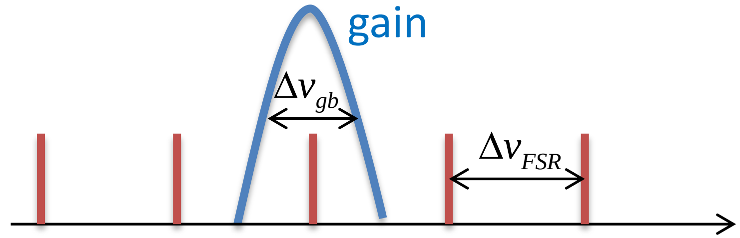
✓ For stable cavity, due to the small volume of TEM00, and the large volume of high-order modes, the latter suffers high loss, and TEM00 oscillates.

# Single longitudinal mode selection

## ❑ Short cavity

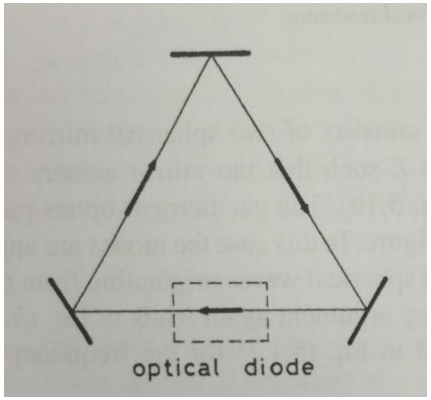
✓ This is appliable when the gain broadening linewidth is not large, like in gas (He-Ne) lasers (1.7 GHz in textbook). Piezo-electric transducer is needed to adjust the cavity length carefully, with accuracy on the order of wavelength.

$$\boxed{\begin{aligned}&\Delta\nu_{FSR} \geq \Delta\nu_{gb}/2 \Rightarrow \\ &L \leq \frac{c}{\Delta\nu_{gb}}\end{aligned}}$$



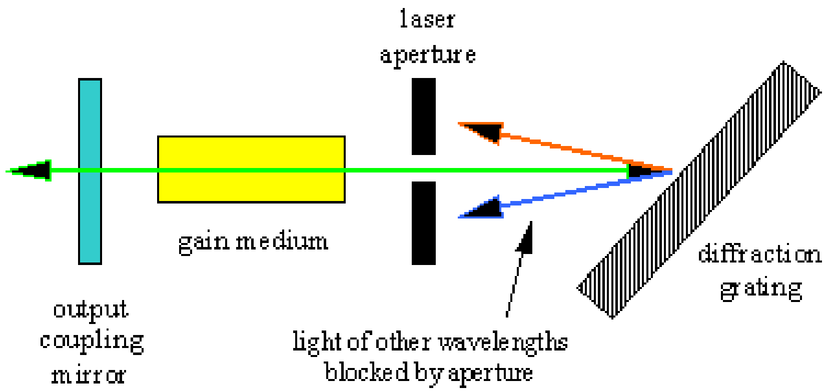

## ❑ Ring cavity with isolator

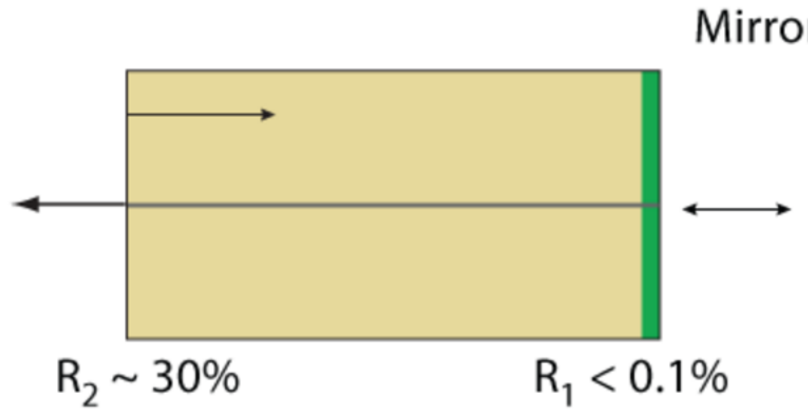
✓ The traveling wave together with homogeneous broadening medium wont form any hole burning effects.

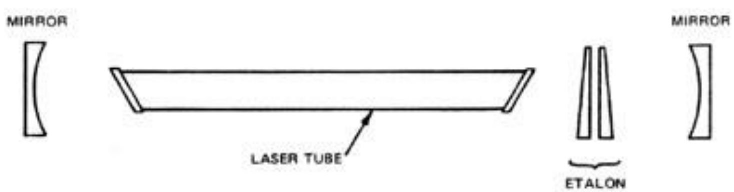


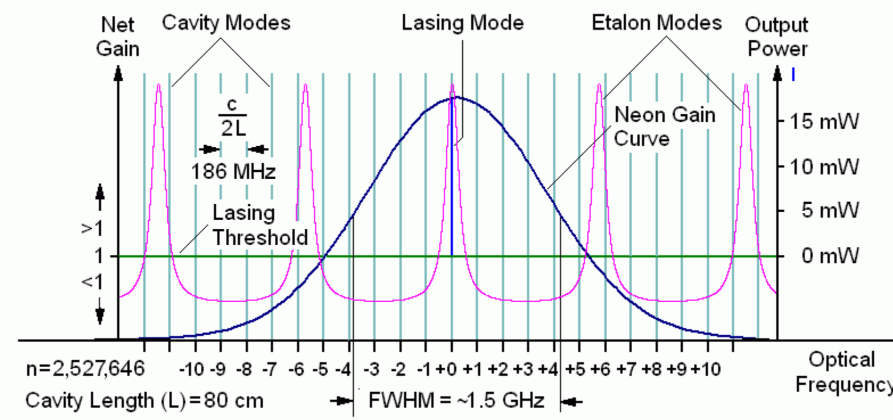
# Single longitudinal mode selection

- Selective mode loss using grating, external cavity, and Fabry-Perot etalon …



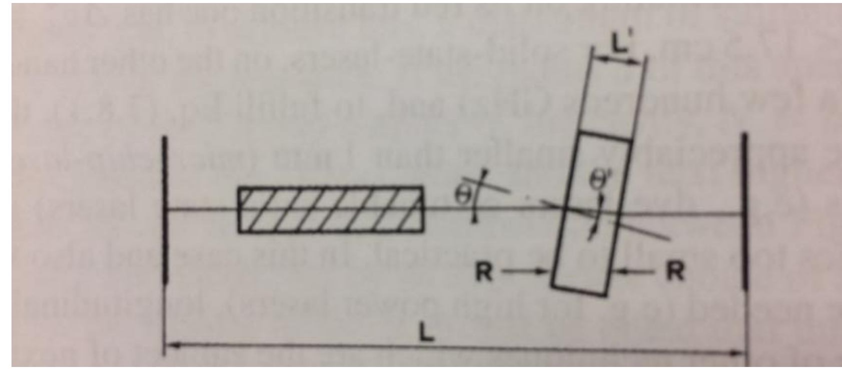




Intracavity Etalon for Line Selection in a Single Mode HeNe Laser

# F-P etalon to select mode

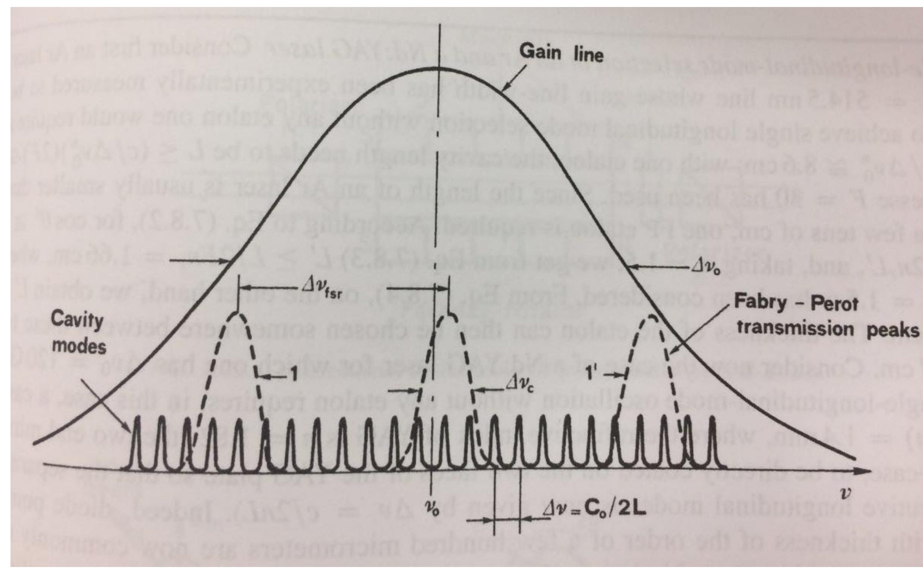
- The transmission maxima of F-P etalon is at

$$\boxed{\nu_m = \frac{mc}{2n_r L'\cos\theta'}}$$




- A small angle adjustment is needed to tune one transmission peak close to the gain peak.

$$\boxed{\theta \approx \theta' \approx 0}$$




$$\Delta\nu_{FSR}^{LS} = \frac{c}{2L};\ \Delta\nu_{FSR}^{ET} = \frac{c}{2n_r L'\cos\theta'};\ \Delta\nu_c^{ET} = \frac{\Delta\nu_{FSR}^{ET}}{F}$$

# F-P etalon to select mode



- The etalon half peak linewidth must be smaller the mode spacing of the laser cavity.

$$\boxed{\frac{1}{2}\Delta v_c^{ET} \le \Delta v_{FSR}^{LS}}$$

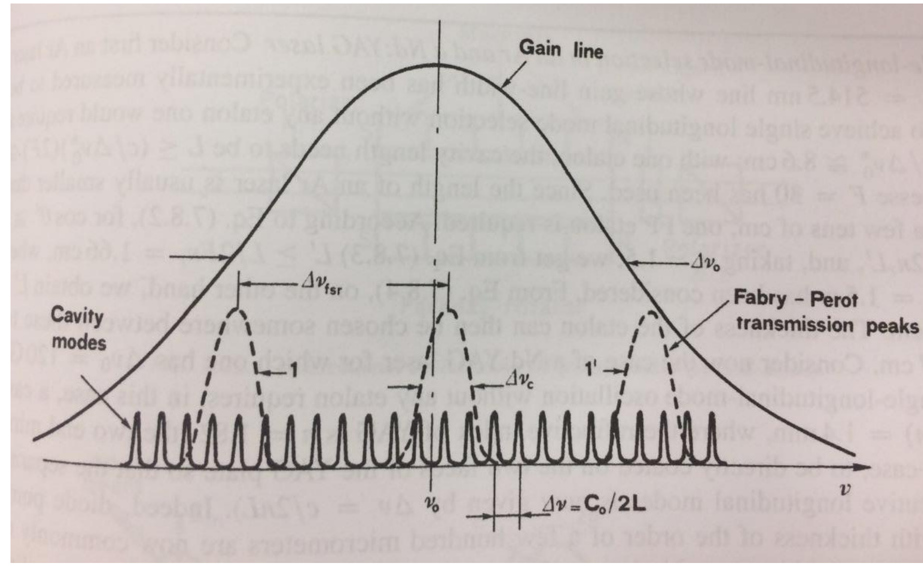
- The etalon FSR must be larger than half gain linewdith

$$\boxed{\Delta v_{FSR}^{ET} \ge \frac{1}{2}\Delta v_{gb}}$$

- Consequently, the requirement

$$\boxed{\frac{1}{2}\Delta v_{gb} \le \Delta v_{FSR}^{ET} \le 2F\Delta v_{FSR}^{LS}}$$

$$\boxed{\frac{L}{2F} \le n_r L' \le \frac{c}{\Delta v_{gb}}}$$




- The laser cavity length requirement can be 2F times larger than that without etalon.

$$\boxed{L \le 2F\frac{c}{\Delta v_{gb}}}$$

Examples 7.8

# Chapter 7_L18



- Frequency pulling effect
- Laser linewidth

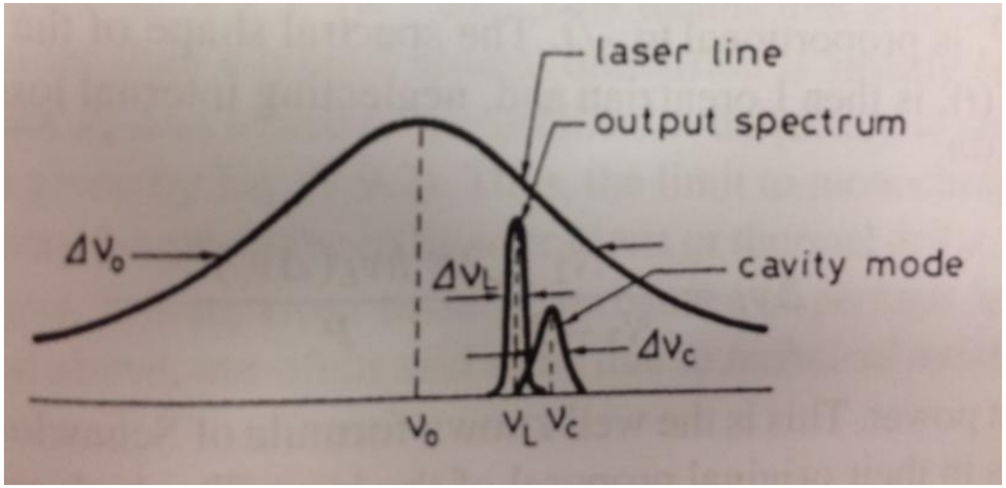
# Frequency pulling effect



❑ Laser frequency pulling effect: for an optical resonator with a cavity mode at $\nu_c$, and a gain medium with the peak at $\nu_0$. The lasing frequency $\nu_L$ will deviate from $\nu_c$, and is pulled toward the gain peak $\nu_0$.

$$\boxed{\nu_L = \frac{\nu_0/\Delta\nu_0 + \nu_c/\Delta\nu_c}{1/\Delta\nu_0 + 1/\Delta\nu_c}}$$

✓ Usually $\Delta\nu_0 >> \Delta\nu_c$, so the frequency pulling is generally very small.



# Laser linewidth

- The linewidth of the lasing mode originates from the spontaneous emission noise, which leads to both random phase fluctuation (dominating) and amplitude fluctuation (small) of the electric field.

$$\boxed{E(t) = A_0 \sin\left[2\pi v_L t + \varphi_{ran}(t)\right]}$$

- The spectral linewdith (Lorentzian shape) of a passive resonant cavity is

$$\boxed{\Delta v_c = \frac{1}{2\pi\tau_p}} \qquad \boxed{\tau_p = \frac{1}{v_g \alpha_T}}$$

- The spectral linewidth (Lorenzian shape) of an active resonant cavity (laser linewidth) is

$$\boxed{\Delta v_L = \frac{1}{2\pi\tau_p^a}} \qquad \boxed{\tau_p^a = \frac{1}{v_g\left(\alpha_T - g\right)}}$$

# Laser linewidth



❑ Above threshold, the « active photon lifetime » is not zero due to the spontaneous emission.

$$\tau_p^a = \frac{1}{v_g\left(\alpha_T - g\right)} > 0 \Rightarrow$$

$$g_{th} < \alpha_T$$

The rate equation

$$\frac{dN_P}{dt} = \mathrm{v_g} g N_P - \frac{N_P}{\tau_p} + \beta \frac{N_2}{\tau_{sp}} = 0 \Rightarrow$$

$$\mathrm{v_g}\left(\alpha_T - g\right) = \beta \frac{N_2}{\tau_{sp}} \frac{1}{N_P} \Rightarrow$$

$$\frac{1}{\tau_p^a} = \frac{1}{N_P} \beta \frac{N_2}{\tau_{sp}}$$

❑ Then, the laser linewidth is

$$\boxed{\Delta v_L = \frac{1}{2\pi N_P} \beta \frac{N_2}{\tau_{sp}}}$$

❑ This is the Schawlow-Townes linewdith, which gives the quantum limit of laser linewidth, 1mHz<typically <1 Hz.

$$\Delta v_L = \frac{N_2}{\Delta N} \frac{2\pi h v_L \left(\Delta v_c\right)^2}{P}$$

❑ The laser linewidth decreases with the lasing power.

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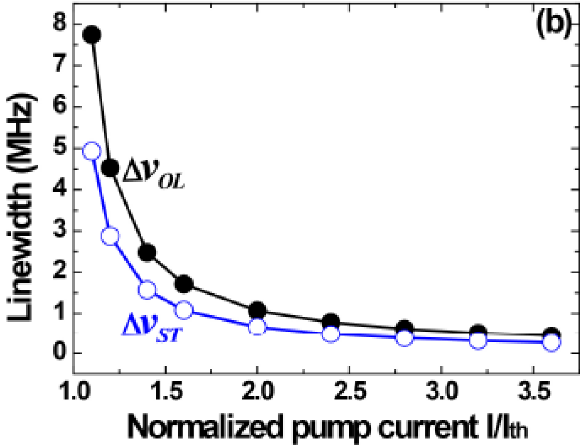
# Laser linewidth



The spectral (optical) linewidth of solid-state lasers, fiber lasers, and gas lasers are on the order of kHz,while the linewidth of semiconductor lasers is on the oder of MHz.

In semiconductor lasers, spontaenous emission noise, on one hand, directly induces phase fluctuations of electric field. On the other hand, induces carrier (electrons and holes) density fluctuations, leading to refractive index changes of the laser medium, and thereby the laser frequency fluctuation, broadening the spectral linewidth. This is called the phase-amplitude coupling effect, and is characterized by the linewidth enhancement factor (Henry factor).

Noise orignating from spontaneous emission and carrier noises is called quantum noise, while that from mechanical vibrations and thermal fluctuations is called technical noise.




$$\boxed{\Delta\nu_{OL} = \left(1+\alpha^2\right)\Delta\nu_{ST}}$$

Examples 7.9

# Chapter 7_L19



- ❑ Laser frequency (phase) noise and fluctuation
- ❑ Laser intensity noise

# Laser frequency/phase noise



- In practice, the laser frequency is not constant, but fluctuates randomly due to the laser noise. Short-term noise (high frequency) is due to the quantum noise (spontaneous emission, carrier noise). Long-term noise (low frequency) is due to technical noise (temperature fluctuation, driven current noise, mirror vibration…).

- Phase noise and frequency noise relation:

$$\boxed{\Delta v_L(t)=\frac{d\varphi_{ran}(t)}{dt}} \qquad \boxed{\Delta v_L(f)=jf\,\Delta\varphi_{ran}(f)}$$

- The laser electric field with random phase:

$$\boxed{E(t)=A_0\sin\left[2\pi v_L t+\varphi_{ran}(t)\right]}$$

- The instantaneous laser frequency:

$$\boxed{v_L(t)=v_L+\Delta v_L(t)}$$

# Laser frequency/phase noise



- The frequency/phase noise is usually characterized by the power spectral density (Fourier transform of autocorrelation function):

$$\boxed{\begin{aligned} S_v(f) &= \int_{-\infty}^{+\infty} < \Delta v(t)\Delta v(t+\tau) > \exp(j\omega\tau)d\tau \\ &\equiv \left|\Delta v(f)\right|^2 \ (\mathrm{Hz}^2/\mathrm{Hz}) \\ S_\varphi(f) &= \int_{-\infty}^{+\infty} < \Delta\varphi(t)\Delta\varphi(t+\tau) > \exp(j\omega\tau)d\tau \\ &\equiv \left|\Delta\varphi(f)\right|^2 \ (\mathrm{rad}^2/\mathrm{Hz}) \end{aligned}}$$

$$\boxed{S_v(f) = f^2 S_\varphi(f)}$$

$$< \Delta v(t)\Delta v(t+\tau) > = \lim_{T\to\infty}\frac{1}{T}\int_0^T \Delta v(t+\tau)\Delta v^*(t)dt$$

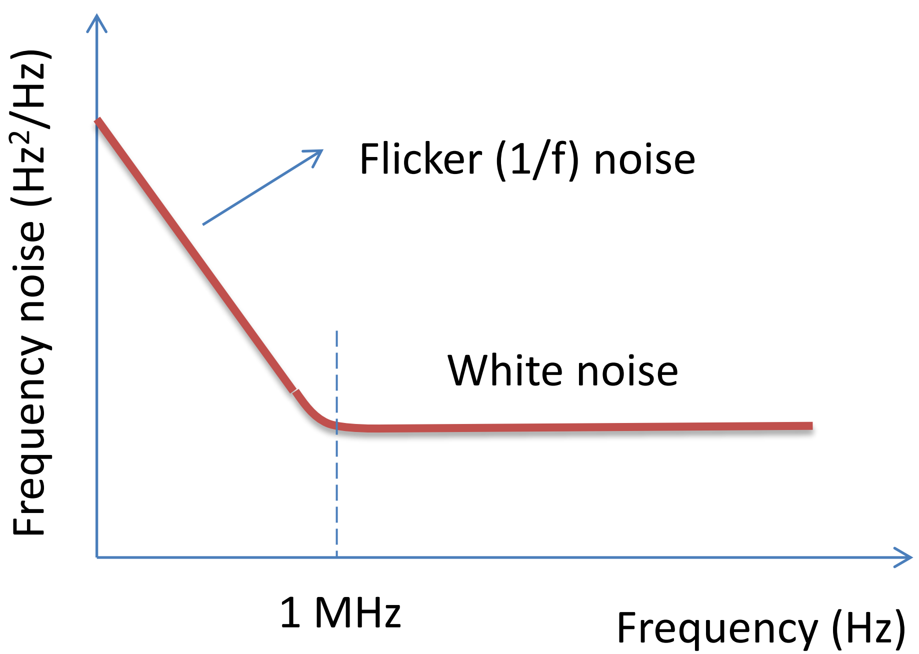
# Laser frequency/phase noise



❑ Spontanenous emission noise is a kind of white noise ($S_v(f)$=const.) in the frequency (electrical) domain, which leads to a Lorentzian-shape spectral (optical) linewidth.  The relation between the laser linewidth and the white frequency noise is

$$\boxed{\Delta v_L = \pi S_v(f)}$$

❑ Besides the white noise due to quantum noise, there is flicker noise at low frequency due to technical noises.



# Frequency fluctuation and stablization



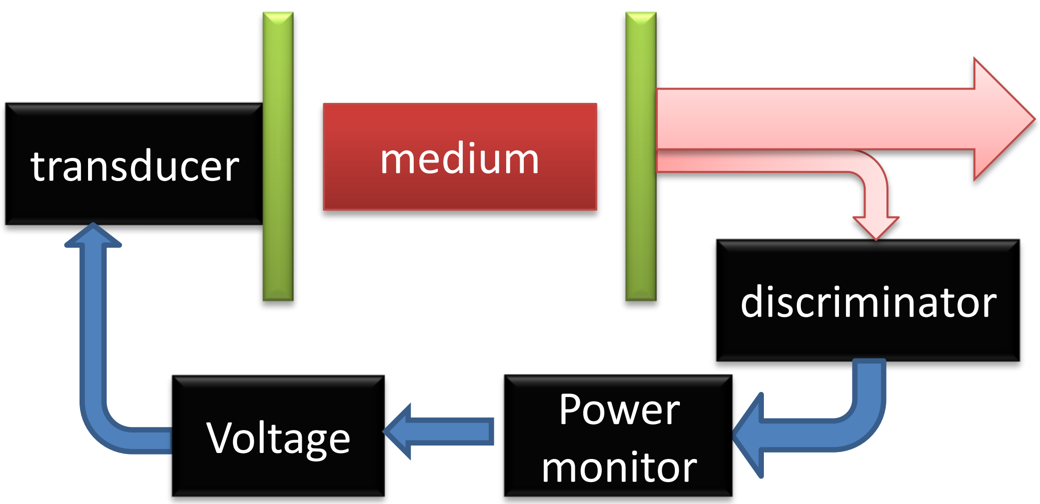
❑ The laser mode frequency fluctuates due to cavity length change and to refractive index change:

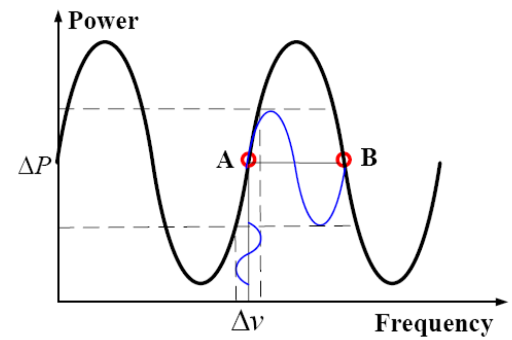
$$\boxed{\begin{aligned} \nu_m &= m\frac{c}{2n_r L} \\ \Delta\nu_m &= -\nu_m\left(\frac{1}{n_r}\Delta n_r + \frac{1}{L}\Delta L\right) \end{aligned}}$$

❑ The frequency stability is usually characterized by $\left|\Delta\nu_m / \nu_m\right|$

Examples 7.10

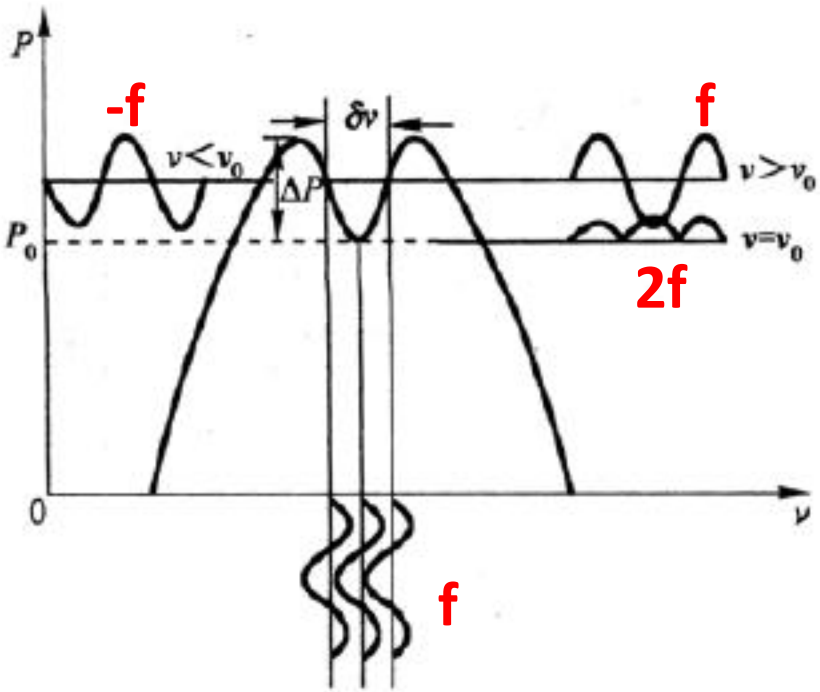
❑ The frequency stablization can be realized by using a frequency discriminator (F-P etalon) together with a piezoelectric transducer



# Frequency stablization using Lamb dip






- ❑ The laser is modulated at a low frequency f.
- ❑ If the lasing frequency $v=v_0$, the lasing power modulation frequency is 2f.

- ❑ If the lasing frequency $v>v_0$, the lasing power modulation frequency is f, and it is in phase with the voltage modulation. Then, the transducer will enlarge the cavity length to move the lasing frequency to $v_0$.
- ❑ If the lasing frequency $v<v_0$, the lasing power modulation frequency is f, and it is out of phase with the voltage modulation. Then, the transducer will reduce the cavity length to move the lasing frequency to $v_0$.

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- Laser frequency (phase) noise and fluctuation
- Laser intensity noise

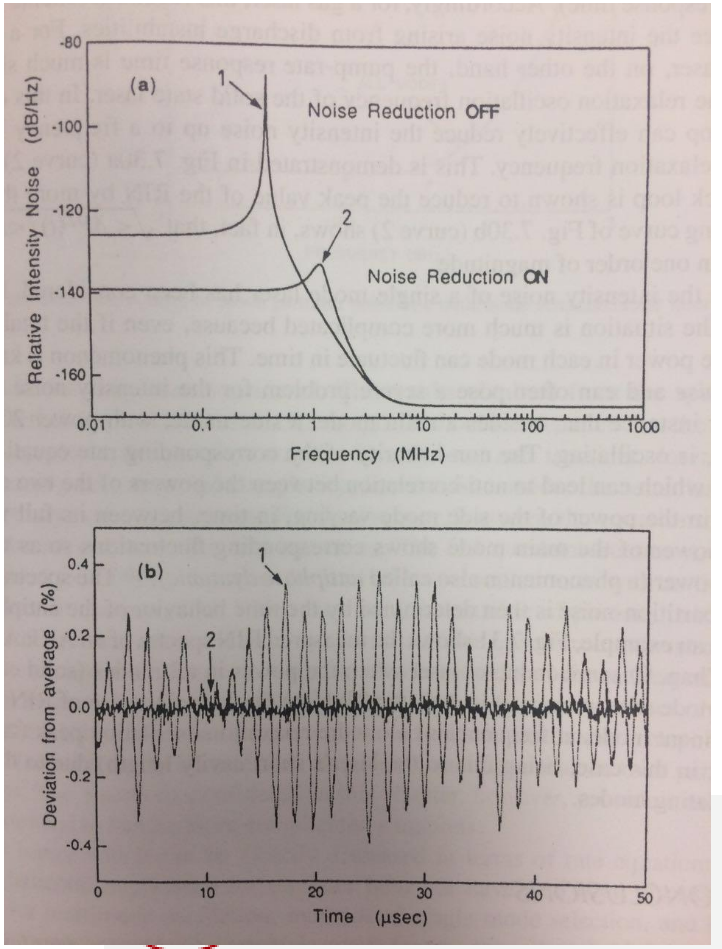
# Laser intensity noise



- The laser intensity noise is usually characterized by the relative intensity noise (RIN), which is defined as the Fourier transform of the autocorrelation of the instantaneous lasing power, with respective to the square of the average lasing power.

$$RIN(f) = \frac{\int_{-\infty}^{+\infty} < P(t)P(t+\tau) > \exp(j\omega\tau)d\tau}{\bar{P}^2}$$

$$\equiv \frac{|P(f)|^2}{\bar{P}^2} \text{(dB/Hz)}$$




- The peak in the RIN spectrum is around the laser's relaxation resonance frequency.
- The RIN can be reduced by applying an optical/optoelectronic feedback loop.

# Homework



7.3

7.4

7.5

7.6

7.8

7.16

7.17

7.18